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DEALING WITH MEASUREMENT NOISE

(A gentle introduction to noise filtering)
by M. Tham

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INTRODUCTION

When measurements are corrupted by random variations, they are said to be affected by **noise**. Since the standard deviation is a measure of spread in a data distribution, these random variations can be characterised by the standard deviation of the measured signal. That is, the larger the standard deviation, the noisier is the measurement. The procedure of reducing or **attenuating** the noise components of a measured signal is commonly known as **filtering**.

There are many different ways to design filters, but the most common ones have their roots in simple averaging.

The purpose of this set of notes is to

- introduce some of the terminology used in the area of signal processing
- show why signal averaging can reduce the effects of noise
- introduce some filtering algorithms that are based on averaging
- show that the low-pass filter that is commonly used in industry is equivalent to one form of averaging filter

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Averaging Filter

Simple averaging can be used to reduce the effects of noise. Suppose we have *n* measurements of a variable *x*. The **standard deviation** of this measured variable can be estimated by:

$$s = \sqrt{\frac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}{n-1}}$$

where $\bar{x}$ is the **mean** or average of the *n* measurements calculated as:

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

The magnitude of *s* is clearly dependent on the measurements, x_i, which in practice is **bounded**, that is it has lower and upper limits. However, *s* is also dependent on the number of measurements made, i.e. the number *n*. Thus for bounded values of x_i, it can be deduced that the larger *n* is, the smaller *s* becomes. In other words, given a noisy but bounded measurement sequence, we can take a large number of readings of the variable and use its average to give a better **estimate** of its true value (provided there is no systematic error or bias in the measurements). This is actually standard procedure in experimental work, where a number of readings are taken at a sampling instant and the average of these readings used as the measurement.

Although it is simple to calculate averages using the formula above, for online applications, it is inefficient both in terms of storage and computational requirements. This is because we need to:

- store *n* data values
- perform *n* additions and 1 division

Given that computer storage and high speed microprocessors are very cheap nowadays, this may not seem to be a problem. However, the load on a process computer can become quite significant when you consider the fact that hundreds, and perhaps even thousands, of measurements are made on a

typical process plant. We therefore need to look at more efficient ways of calculating averages.

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Moving Average Filter

A slight improvement in computational efficiency can be achieved if we perform the calculation of the mean in a **recursive** fashion. A recursive solution is one which depends on a previously calculated value. To illustrate this, consider the following development:

Suppose that at any instant *k*, the average of the latest *n* samples of a data sequence, x_i, is given by:

$$\bar{x}_k = \frac{1}{n}\sum_{i=k-n+1}^{k} x_i$$

Similarly, at the previous time instant, *k-1*, the average of the latest *n* samples is:

$$\bar{x}_{k-1} = \frac{1}{n}\sum_{i=k-n}^{k-1} x_i$$

Therefore, $\bar{x}_k - \bar{x}_{k-1} = \frac{1}{n}\left[\sum_{i=k-n+1}^{k} x_i - \sum_{i=k-n}^{k-1} x_i\right] = \frac{1}{n}\left[x_k - x_{k-n}\right]$

which on rearrangement gives: $\bar{x}_k = \bar{x}_{k-1} + \frac{1}{n}\left[x_k - x_{k-n}\right]$

This is known as a **moving average** because the average at each *k*'th instant is based on the most recent set of *n* values. In other words, at any instant, a **moving window** of *n* values are used to calculate the average of the data sequence (see Figure 1).

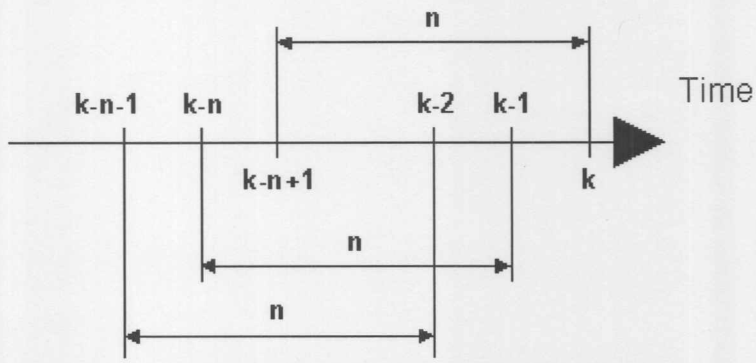

Figure 1 Moving Window of *n* Data Points

When used as a filter, the value of $\bar{x}_k$ is taken as the filtered value of x_k. The expression is a recursive one, because the value of $\bar{x}_k$ is calculated using its previous value, $\bar{x}_{k-1}$, as reference.

Compared to simple averaging, it can be seen that we need only perform 1 division, 1 addition and 1 subtraction operation. This is always the case, regardless of the number of data points (*n*) we consider. However, calculating the current filtered value requires the use of x_{k-n}, i.e. the measurement *n* time-steps in the past.

This means that:

1. the filtering cannot be initiated reliably until *n* measurements have been made, and
2. we need to store the value of x_{k-n} which, depending on the way the algorithm is coded, may require up to *n* storage locations.

Additionally, the technique places equal emphasis on all data points. Thus a value in the past will have the same influence as a more current measurement when calculating the filtered signal. This may be a desirable feature when the mean value of the measurement is almost constant, but not when the signal has a trend as most process measurements do.

These problems can however, be reduced by generating the filtered value in a slightly different manner.

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Exponentially Weighted Moving Average Filter

The moving average filter regards each data point in the data window to be equally important when calculating the average (filtered) value. In dynamic systems, however, the most current values tend to reflect better the state of the process. A filter that places more emphasis on the most recent data would therefore be more useful. Such a filter can be designed by following the procedure used in developing the moving average filter. As before, the starting point is the mean expressed as:

$$\bar{x}_k = \frac{1}{n}\sum_{i=k-n+1}^{k} x_i$$

But in this case, *consider also the mean with one additional point*

$$\bar{x}_{k+1} = \frac{1}{n+1}\sum_{i=k-n+1}^{k+1} x_i = \frac{1}{n+1}\left[x_{k+1} + \sum_{i=k-n+1}^{k} x_i\right]$$

Since $\sum_{i=k-n+1}^{k} x_i = n\bar{x}_k$, therefore,

$$\bar{x}_{k+1} = \frac{1}{n+1}\left[x_{k+1} + n x_k\right] = \left(\frac{1}{n+1}\right)x_{k+1} + \left(\frac{n}{n+1}\right)\bar{x}_k$$

By shifting the time index back one time-step, we obtain the corresponding expression for $\bar{x}_k$ as:

$$\bar{x}_k = \left(\frac{1}{n+1}\right)x_k + \left(\frac{n}{n+1}\right)\bar{x}_{k-1}$$

To simplify the notation, let $\alpha = \frac{n}{n+1}$, which implies that $(1-\alpha) = \left(\frac{1}{n+1}\right)$.

We can write the filter as:

$$\bar{x}_k = \alpha\bar{x}_{k-1} + (1-\alpha)x_k$$

This expression is known as the *Exponentially Weighted Moving Average Filter*. When used as a filter, the value of $\bar{x}_k$ is again taken as the filtered value of x_k. Notice that now, calculation of $\bar{x}_k$ does not require storage of past values of *x*, and that only 1 addition, 1 subtraction, and 2

multiplication operations are required.

The value of the **filter constant**, α, dictates the **degree of filtering**, i.e. how strong the filtering action will be. Since $n \geq 0$, this means that $0 \leq \alpha < 1$. When a large number of points are being considered, $\alpha \to 1$, and $\bar{x}_k \to \bar{x}_{k-1}$. This means that the degree of filtering is so great that the measurement does not play a part in the calculation of the average! On the other extreme, if $n \to 0$, then $\bar{x}_k \to x_k$ which means that virtually no filtering is being performed.

The Exponentially Weighted Moving Average filter places more importance to more recent data by discounting older data in an exponential manner (hence the name). This characteristic can be illustrated simply by describing the current average value in terms of past data. For example, since

$$\bar{x}_k = \alpha \bar{x}_{k-1} + (1-\alpha) x_k,$$

then

$$\bar{x}_{k-1} = \alpha \bar{x}_{k-2} + (1-\alpha) x_{k-1}$$

Therefore,

$$\bar{x}_k = \alpha \bar{x}_{k-1} + (1-\alpha) x_k = \alpha\left[\alpha \bar{x}_{k-2} + (1-\alpha) x_{k-1}\right] + (1-\alpha) x_k$$

i.e. $\bar{x}_k = \alpha^2 \bar{x}_{k-2} + \alpha(1-\alpha) x_{k-1} + (1-\alpha) x_k$

But $\bar{x}_{k-2} = \alpha \bar{x}_{k-3} + (1-\alpha) x_{k-2}$,

Therefore,

$$\begin{aligned}\bar{x}_k &= \alpha^2\left[\alpha x_{k-3} + (1-\alpha) x_{k-2}\right] + \alpha(1-\alpha) x_{k-1} + (1-\alpha) x_k \\ &= \alpha^3 \bar{x}_{k-3} + \alpha^2(1-\alpha) x_{k-2} + \alpha(1-\alpha) x_{k-1} + (1-\alpha) x_k\end{aligned}$$

If we keep on expanding $\bar{x}$ terms on the right hand side, we will see that the contribution of older values of x_i are weighted by increasing powers of α. Since α is less than 1, the contribution of older values of x_i becomes progressively smaller. The weighting on x_i may be represented graphically by the following plot:

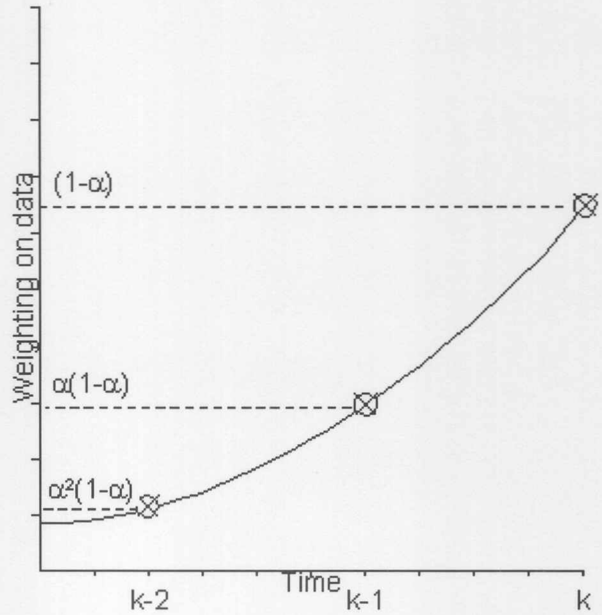

Figure 2. Exponential weighting effect

What this means is that in calculating the filtered value, more emphasis is given to more recent measurements.

The Exponentially Weighted Moving Average filter is arguably the most commonly used noise reduction algorithm in the process industries. However, it is known commonly by a another name; one that has its roots in electrical circuitry that are used to produce smooth electrical signals.

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The 1st-order Low-pass Filter

The Exponentially Weighted Moving Average filter is identical to the discrete *first-order low-pass filter* - a commonly available feature in most process computers. The objective of this section is to show their equivalence.

Consider the Laplace transfer function of a first-order low-pass filter, with time constant τ_f.

$$\frac{\bar{x}(s)}{x(s)} = \frac{1}{1+\tau_f s}$$

which relates the filtered signal $\bar{x}(s)$ to the measurement $x(s)$. This has the following time domain equivalent:

$$\tau_f \frac{d\bar{x}(t)}{dt} + \bar{x}(t) = x(t)$$

(Note: this differential equation can also be used to describe the input and the output behaviour of an electrical RC-circuit)

Now, the differential equation can be discretised using the approximation:

$$\frac{d\bar{x}(t)}{dt} \approx \frac{\bar{x}_k - \bar{x}_{k-1}}{T_s}$$

where T_s is the interval between each measurement, i.e. the sampling interval. Thus the differential equation representing the first-order low pass filter is converted to:

$$\tau_f \frac{\bar{x}_k - \bar{x}_{k-1}}{T_s} + \bar{x}_k = x_k$$

Simplification and re-arrangement gives:

$$\bar{x}_k = \left(\frac{\tau_f}{\tau_f + T_s}\right)\bar{x}_{k-1} + \left(\frac{T_s}{\tau_f + T_s}\right)x_k$$

By letting

$$\alpha = \left(\frac{\tau_f}{\tau_f + T_s}\right) \Rightarrow (1-\alpha) = \left(\frac{T_s}{\tau_f + T_s}\right),$$

then

$$\bar{x}_k = \alpha\bar{x}_{k-1} + (1-\alpha)x_k$$

which is identical to the Exponentially Weighted Moving Average filter.

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Choice of Filter Constants

The degree of noise attenuation using the Exponentially Weighted Moving Average and the first-order low-pass filter is determined by the parameter α.

With the Exponentially Weighted Moving Average algorithm, α is determined by, n, the number of data points being considered in the calculation, i.e.

$$\alpha = \frac{n}{n+1}$$

In the first-order low-pass filter, α is determined by τ_f, the filter time-constant, i.e.

$$\alpha = \left(\frac{\tau_f}{\tau_f + T_s}\right)$$

Since $\alpha = \frac{n}{n+1}$ and $\alpha = \left(\frac{\tau_f}{\tau_f + T_s}\right)$, it is obvious that n and τ_f must be related. After some algebraic manipulation, the exact relationship can be worked out to be

$$\tau_f = nT_s.$$

Based on the previous discussion on the effects of α on the degree of filtering, it follows that the larger the filter time-constant, the higher the degree of filtering. This is illustrated in the diagrams below.

Figure 3 shows the results of sending a signal of unit step (dashed line) to a low-pass filter with different values of α, while Figure 4 shows the results when a noisy step input is sent to the same set of filters.

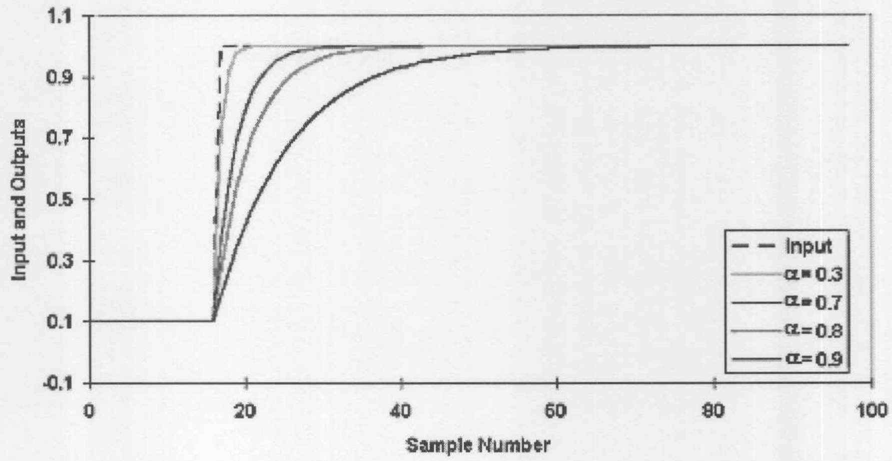

Figure 3 Outputs of first-order filters to a noise free step input

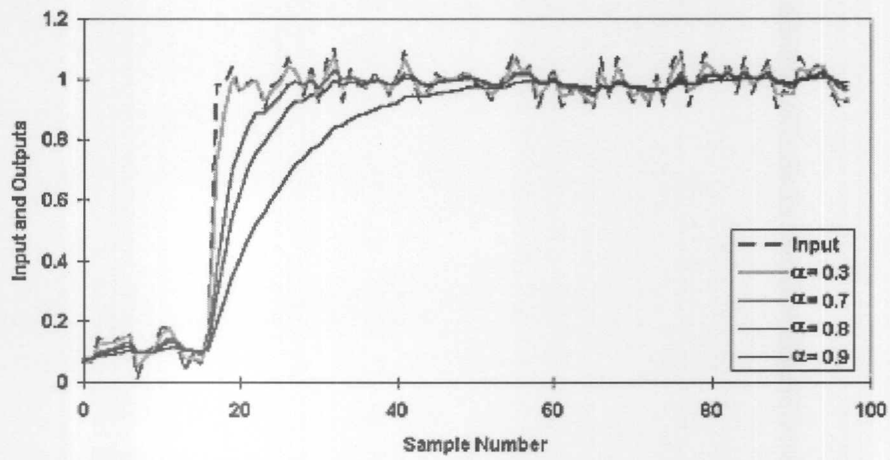

Figure 4 Outputs of first-order filters to a noisy step input

These plots reveal that noise attenuation is good when α is set to a high value, but this is achieved at the expense of large lags on the filtered signal. If the filtered signals are used for feedback control, then excessive lags can severely degrade the stability margins of the closed loop system. On the other hand, the filter with a value of $\alpha = 0.3$ hardly removed the noise. From experience, a value of $\alpha = 0.7$ seems to give good performances, and this is equivalent to choosing $n \approx 2$ or $\tau_f \approx 2T_s$.

The fact is that a compromise has to be made when selecting a value for α and is the principal noise filter design objective; achieving sufficient noise attenuation with minimum processed signal lags. That however, is another story.

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Frequency Characteristics

Low-pass filters allow the low frequency components of an input signal to pass through while attenuating (reducing) high frequency components, hence the term **low-pass**. There are other filters, such as **band-pass** and **high-pass** filters, where the classification is based on the frequency ranges that the filter allows to pass through.

Measurement noise fall into the high frequency range of the signal spectrum, while the underlying process signal usually lies towards the low frequency end. Thus filters that are used to remove noise from measurements are of the low-pass types. The exponentially weighted moving average filter (or equivalently the 1st-order low-pass filter) are but one of many possible types of low-pass filters.

A signal can be decomposed into components of different frequencies. Thus, one method of examining the capabilities of filters is to look at what happens to inputs of different frequencies when they are passed through the filter. There are two features of the output signals that we can investigate:

1. what are the differences between the magnitudes of the input and the output signals (given by the **amplitude ratio**) and
2. whether there is a time lag between input and output signals (given by the **phase-shift**)

Plots of amplitude-ratio and phase-shift against frequency gives the **frequency response** of the system. For simplicity, in this set of introductory notes, we will consider only the amplitude ratio characteristics of low-pass filters.

The amplitude ratio can be regarded as a frequency dependent gain of the filter. The job of the low-pass filter is to filter out high frequency components, therefore its amplitude ratio should be low at high frequencies. At low frequencies, the low-pass filter should allow the input signal to pass through undistorted, and so its amplitude ratio at the low frequency end of the signal spectrum should be unity. The red lines in the figure below show the frequency dependent amplitude ratio plots of 3 first-order low-pass filters, with time-constants 10, 20 and 30.

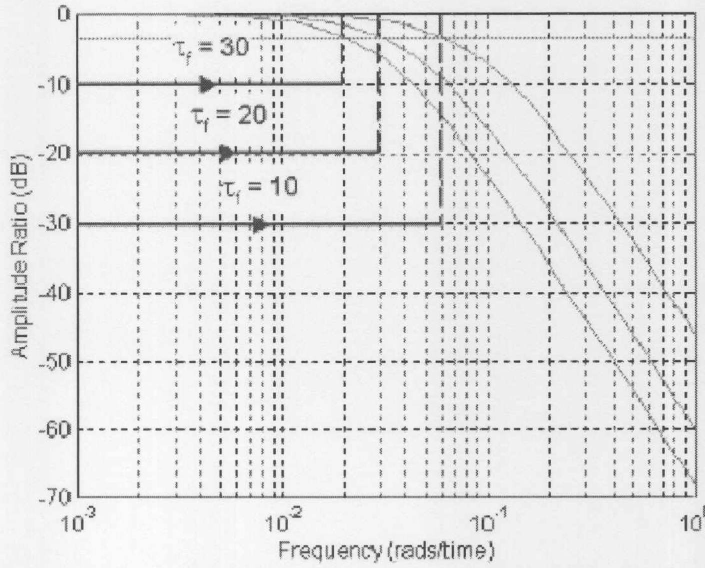

Figure 5. Amplitude ratio plots of 1st-order low-pass filters with differen time-constants

Note that the amplitude ratios in the above figure are expressed in terms of **decibels** (dB).

To convert a number, *x*, to dB, simply apply a log and multiply the result by 20, that is, $20\log_{10}(x)$. Therefore 0 dB is equivalent to an amplitude ratio of 1, while a large negative dB indicates that the amplitude ratio is very small.

From Figure 5, we can see that first-order filters are capable of attenuating high frequency components. In particular, *the larger the time-constant of the filter, the higher the degree of filtering*. This is easily discerned by examining the amplitude ratios of the three filters at any particular frequency (see the three vertical, dashed blue lines in Figure 5).

A measure of the efficiency of a filter is its **bandwidth**. This is defined as the frequency range of a signal that a filter allows to pass through with minimal attenuation; signal attenuation is considered to be significant when the amplitude ratio is less then -3dB, (approximately 0.7 as a ratio). The -3dB line is the horizontal magenta coloured line in Figure 5, from which it can be seen that the *bandwidth of a filter decreases with increasing values of filter time-constants*.

Low-pass filters need not be limited to first-order types. Figure 6 shows the frequency responses of 3 low-pass filters with the following Laplace transfer functions:

First-order low-pass filter $\dfrac{1}{1+10s}$

Second-order low-pass filter	$\frac{1}{(1+10s)^2}$
Third-order low-pass filter	$\frac{1}{(1+10s)^3}$

The second and third-order filters are respectively, implemented by placing 2 and 3 first-order filters in series.

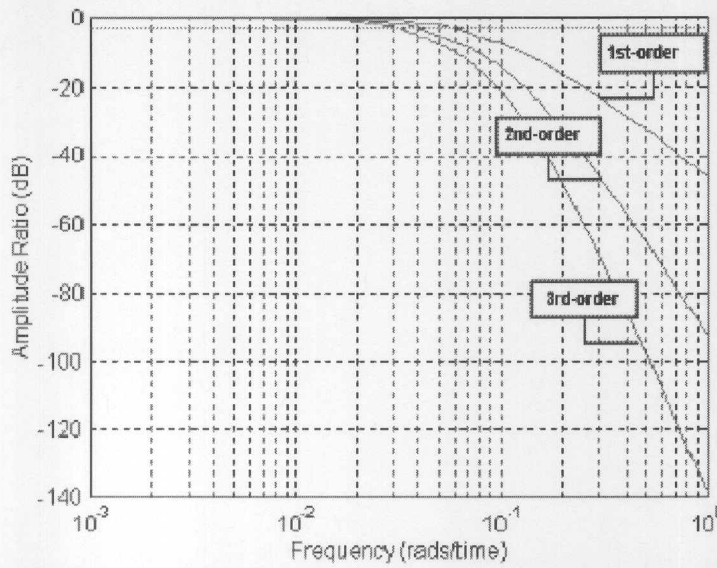

Figure 6. Amplitude ratio plots of 1st, 2nd and 3rd-order low-pass filters

We can see from Figure 6 that as the order of the filter increases, the slopes of the respective amplitude ratio plots becomes steeper. What this indicates is that *higher order low-pass filters provide higher rates of signal attenuation*. From the -3dB line (magenta line), it is also clear that *the bandwidth of a filter decreases with increasing order*. Thus a higher degree of filtering can be achieved by employing higher order filters.

Theoretically, we can design a filter such that its amplitude ratio beyond some frequency, ω_c, is a vertical line, while the amplitude ratio below this threshold frequency is horizontal at 0 dB. A filter with this kind of frequency response characteristics is called an **ideal low-pass filter**, and ω_c is called the **cut-off frequency**; that is any signal with frequency components higher than this will be completely removed by the filter. Thus, the *bandwidth of the ideal filter is equal to its cut-off frequency.*

As mentioned previously, detrimental lags are introduced into the processed signal by low-pass filters. This aspect can also be studied by plotting the filter's frequency dependent phase shift characteristics. However, the effects are not as graphic as time-

response plots, and so phase-shift plots are not covered in this set of introductory notes. Those who are interested can find further details in standard signal processing texts, or read the appropriate section of the notes on Applications of Frequency Response (requires the free Acrobat Reader).

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